Nonlinear inductive response of high temperature superconducting films measured by the mutual inductance technique

J. H. Claassen[a)]
Naval Research Laboratory, Washington, DC 20375-5343

James C. Booth, J. A. Beall, D. A. Rudman, L. R. Vale, and R. H. Ono
National Institute of Standards and Technology, Boulder, Colorado 80303

(Received 5 February 1999; accepted for publication 3 May 1999)

The dependence of the penetration depth λ on current density J in an unpatterned superconducting film can be measured with a pair of small coaxial coils positioned on opposite sides of the film. Mutual inductance measurements in this configuration with a direct-current (dc) current component in one of the coils provide a means to determine $\lambda(J)$. It is possible to separate out the effects of heating by initially trapping a persistent current in the film. The nonlinear (current-dependent) coefficients of $\lambda(J)$ measured by this ''dc'' technique agree well with those measured by third harmonic generation in coplanar waveguide transmission lines at 5 GHz. This nondestructure technique could be used to screen films before incorporating them into circuits sensitive to nonlinear effects. [S0003-6951(99)03226-X]

In microwave circuits using high T_c superconductor (HTS) thin films an important issue is the occurrence of nonlinear effects, such as intermodulation distortion and harmonic generation.[1] These effects can be particularly detrimental for applications with large current densities, such as high Q filters or resonators, or high power devices. Many observed nonlinear effects can be modeled by assuming the presence of a nonlinear inductance which arises due to a current-dependent superconducting penetration depth of the form:[2]

$$\lambda^2(T,J)=\lambda^2(T,O)[1+(J/J_0)^2]. \tag{1}$$

Low power third-order intermodulation measurements of resonators made from films of $Tl_2Ba_2CaCu_2O_y$ (TBCCO),[3] and third harmonic generation measurements of coplanar waveguide transmission lines made from $YBa_2Cu_3O_y$ (YBCO) thin films[4] have been quantitatively described assuming this form for $\lambda(J)$. The parameter J_0 can then be used as a figure of merit for quantifying the nonlinear response of a particular material (a larger value of J_0 yields a smaller nonlinear response). However, the values obtained for J_0 from such nonlinear measurements are meaningful only as long as Eq. (1) is a good approximation for $\lambda(J)$.

In this letter we show that a low frequency, noncontacting measurement of unpatterned HTS films can be used to determine $\lambda(J)$. Results of this measurement technique confirm the suitability of Eq. (1) for describing nonlinear effects in YBCO samples, and the values obtained for J_0 agree well with those inferred from microwave measurements. In addition to allowing films to be vetted as to their suitability for use in microwave circuits prior to patterning, the values of $\lambda(J)$ obtained by this technique should also enable the prediction of the nonlinear response of patterned microwave devices.[4]

[a)]Electronic mail: john.claassen@nrl.navy.mil

The method used here to extract $\lambda(J)$ has been described previously.[5] In brief, the penetration depth is measured using a pair of coils on a common axis positioned on opposite sides of the film. As long as the coil dimensions are sufficiently small compared to the lateral extent of the film, the mutual inductance between them can be shown[6–8] to be proportioanl to $\lambda/\sinh(d/\lambda)$, where d is the film thickness. Since the proportionality factor can be calculated from the coil parameters, a mutual inductance measurement yields the absolute value of the penetration depth with resolution that can be less than 0.1 nm in practice.[6]

A current in either of the coils induces a screening current in the superconducting film. Thus, if the drive coil has a dc current as well as the alternating-current (ac) component required to measure mutual inductance, the penetration depth of the film in te presence of a dc current is obtained. In order to generate the largest film current for a given coil current, the coil closest to the film should be used as the drive coil.[9] The screening current induced in the film has a strong radial dependence with a maximum J_{max} centered about the median coil radius. It has been argued[5] that the measured penetration depth is mostly governed by J_{max}; that is, the measured dependence $\lambda(J_{max})$ is quite close (but not necessarily identical) to the actual $\lambda(J)$ dependence of the film. In this letter we represent the current density in the film by $J_{max}=kI_{coil}/d$, where d is the film thickness and the parameter k is calculated from the coil geometry to be $k=1.6\times10^5$/m. If Eq. (1) applies, the change in penetration depth $\Delta\lambda=\lambda(T,J)-\lambda(T,O)$ resulting from a dc component of coil current is then given by $\Delta\lambda=(\lambda/2)(k/dJ_0)^2(I_{coil})^2$, where $\lambda=\lambda(T,O)$ and we have assumed $J/J_0\ll1$ in Eq. (1).

An important practical difficulty is that power dissipated by a dc current in the drive coil heats up the film, which in turn increases the penetration depth by virtue of its temperature dependence. It is not possible to distinguish this effect from the dependence in Eq. (1) of λ on current, since both depend quadratically on coil current. This was not a problem for the data reported in Ref. 5, which were taken at low

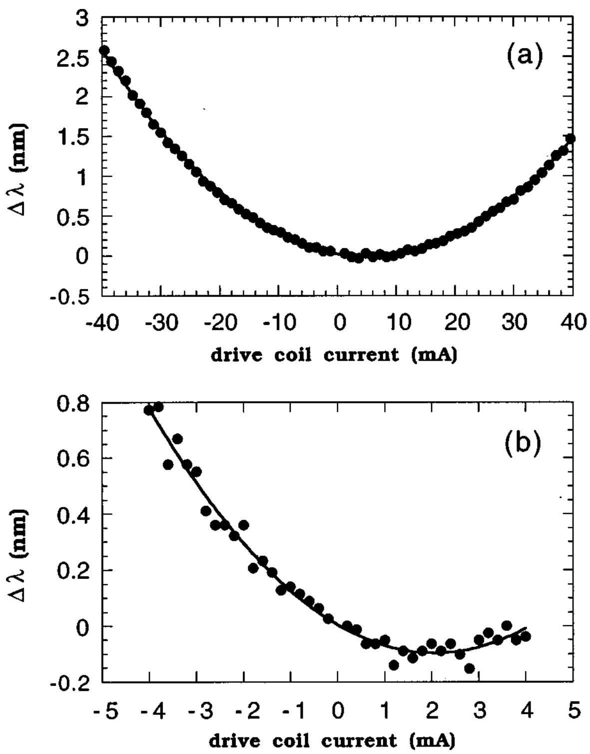

FIG. 1. The change of penetration depth λ as a function of dc current in the drive coil at 78 K, for (a) a 450 nm YBCO film cooled with $I_{trap} = 25$ mA in the drive coil, and (b) a 50 nm YBCO film cooled with $I_{trap} = 2.5$ mA in the drive coil. The lines are fits to the form $a(I_{coil})^2 + b(I_{coil} - I_{trap})^2 + c$.

temperatures and with low coil current. However, for applications of HTS materials, there is an interest in measurements around 77 K. Here the heating problem can be significant since the coil resistance is larger, λ has a fairly strong temperature dependence, and the coil current required to observe an effect is larger.

For measurements of HTS materials, we use a technique based on the large pinning in HTS films to separate the effects of heating and film current on λ. The sample is cooled through its transition with a current I_{trap} in the drive coil. The magnetic field due to this coil current becomes frozen in the film as trapped flux vortices below the transition, but no macroscopic screening currents develop. This is a well-known result of cooling a superconductor in a field and is a consequence of the large field energy required to expel fields in this geometry. The film current subsequently is given by $J_{max} = k(I_{coil} - I_{trap})/d$, and we expect a current-induced dependence of the penetration depth:[10]

$$\Delta\lambda = (\lambda/2)(k/dJ_0)^2(I_{coil} - I_{trap})^2. \tag{2}$$

The heating effect as before will depend on power dissipated in the coil as $\Delta\lambda_{heating} \sim (I_{coil})^2$, where the proportionality depends on the thermal coupling between coil and film, coil resistance, etc.

Figure 1(a) shows a measurement of $\Delta\lambda$ as a function of coil current for a 450-nm-thick YBCO film at 78 K, which had been cooled with a coil current $I_{trap} = 25$ mA. The mutual inductance at each dc current level was measured after thermal equilibration with an ac current component of 2.5 mA at a frequency of 10 kHz. These data can be well fitted to a dependence $\Delta\lambda = a(I_{coil})^2 + b(I_{coil} - I_{trap})^2$, with $I_{trap} = 25$ mA. The coefficient of the current-induced term (b) is around 4 times smaller than the heating contribution (a), yet the signal-to-noise ratio is good enough to determine b with an uncertainty of $\sim 10\%$. We can then extract a value of $J_0 = (k/d)(\lambda/2b)^{1/2} = 2.8 \times 10^7$ A/cm^2. The heating effect in this example represents the most extreme case among films we have examined. Consideration of Eq. (2) shows that the coefficient b will be grater for samples with smaller thickness or smaller J_0, while the heating effect (coefficient a) is proportional to $d\lambda/dT$ which is independent of film thickness. In Fig. 1(b) we show data from a 50 nm YBCO film, cooled with a coil current $I_{trap} = 2.5$ mA. The fitted value of b is around 6 times greater than a, yielding a more reliable result for $J_0 = 3.1 \times 10^7$ A/cm^2.

In samples with $d > \lambda$, the current density is not uniform across the film thickness. For this case we define an average current density $J_{ave} = kI_{coil}/d$, and the mutual inductance measurement gives an average penetration depth (λ_{ave}). It is just these average quantities that are relevant in predicting the nonlinear response in a microwave circuit. However, we should not expect the measured $\lambda_{ave}(J_{ave})$ dependence of a thick film to exactly mirror the intrinsic dependence of the material.

To compare the dc measurement of J_0 with an rf measurement, we prepared three separate series of YBCO films by laser deposition. The series differed by either film thickness or deposition conditions. One film from each series was measured as described above, while its companions were patterned into a series of coplanar waveguide transmission lines and tested for third harmonic generation at frequencies of 3 and 5 GHz. Analysis of these results by the method of Ref. 4 yielded values of the nonlinear scaling current $(J_0)_{rf}$ which were shown not to depend on line length or dimensions. We found remarkable agreement between the rf and dc values of J_0: 29, 28, and 18 MA/cm^2, compared to 31, 35, and 19 MA/cm^2. The first two values are for thicknesses of 50 and 400 nm but otherwise identical growth conditions, demonstrating that J_0 does not depend strongly on film thickness.

Measurements by the dc technique were also performed on a second series of commercially obtained HTS films prepared by off-axis sputtering. A 450-nm-thick YBCO film on a buffered LAO substrate yielded a value $J_0 = 28$ MA/cm^2 that is quite comparable with the best of the PLD samples. Figure 1(a) shows the data from this film. A second 350-nm-thick YBCO film on a buffered sapphire substrate yielded a substantially lower value of $J_0 = 5$ MA/cm^2. Results from the third sample, 650-nm-thick TBCCO on a LAO substrate, are given in Fig. 2. In this case the nonlinear response is much greater than the estimated heating effect. For comparison, we also show the deduced $\Delta\lambda(J)$ data from the samples whose raw data are shown in Fig. 1.

For the TBCCO film, the $\lambda(J)$ dependence is quadratic only in a very small range of current. Recent measurements of intermodulation in a microstrip resonator[11] fit a dependence $\Delta\lambda \sim |J|^{12}$ better than $\Delta\lambda \sim J^2$. The resonator was fabricated from TBCCO films obtained from a different supplier than that reported here and thus cannot be directly compared. Nevertheless, it is striking that the inferred $\Delta\lambda(J) \sim |J|$ dependence in Ref. 11 is qualitatively similar to that observed in Fig. 2.

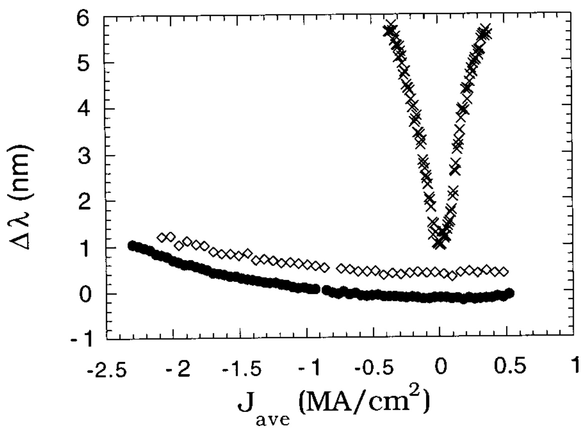

FIG. 2. The change of penetration depth λ as a function of film current density J_{ave} for the samples of Fig. 1(a) (solid circles), Fig. 1(b) (diamonds), and the TBCCO sample (crosses). In the first two cases the fitted heating contribution to the raw data has been removed, and the current density calculated as $J_{\mathrm{ave}}=k(I_{\mathrm{coil}}-I_{\mathrm{trap}})/d$. In the last case the estimated heating effect is negligible and there was no trapped current. The data are offset vertically for clarity.

The observed values of J_0 for the YBCO samples can be compared with the Bardeen–Cooper–Schrieffer (BCS) calculation for the contribution of the quasiparticle backflow to the supercurrent. This predicts a quadratic dependence as in Eq. (1), and J_0 has been calculated for both the usual *s*-wave gap symmetry as well as *d* wave.[2] At a reduced temperature $T/T_c=0.9$ the calculation gives $J_0=14.5\times 10^7$ and 7.5×10^7 A/cm^2 for the two cases, respectively, where it was assumed that the pair breaking critical current was 3×10^8 A/cm^2.These theoretical values are considerably greater than what is measured in even the best YBCO samples, suggesting that the observed values of J_0 may be attributed to extrinsic properties such as planar defects.

In summary, the mutual inductance measurement can predict the inductive nonlinearity of a HTS film in a microwave circuit prior to the patterning and contacting. In addition the measurement gives the penetration depth, which is needed to calculate the nonlinear response and figures importantly in the surface resistance. Heating effects can be distinguished from a quadratic current dependence of the penetration depth by trapping a current in the film. The observed nonlinearity in the best films is significantly larger than predicted by BCS theory.

The portion of this work performed at the National Institute of Standards and Technology was supported in part by the Office of Naval Research.

[1] C. Wilker, Z.-Y. Shen, P. Pang, W. L. Holstein, and D. W. Face, IEEE Trans. Appl. Supercond. **5**, 1665 (1995).
[2] T. Dahm and D. J. Scalapino, Appl. Phys. Lett. **69**, 4248 (1996); T. Dahm and D. J. Scalapino, J. Appl. Phys. **81**, 2002 (1997).
[3] Balam A Willemsen, K. E. Kihlstrom, T. Dahm, D. J. Scalapino, B. Gowe, D. A. Bonn, and W. N. Hardy, Phys. Rev. B **58**, 6650 (1998).
[4] James C. Booth, J. A. Beall, D. A. Rudman, L. R. Vale, and R. H. Ono, J. Appl. Phys. (to be published).
[5] J. H. Claassen, S. Adrian, and R. J. Soulen, Jr., IEEE Trans. Appl. Supercond. (to be published).
[6] J. H. Claassen, M. L. Wilson, J. M. Beyers, and S. Adrian, J. Appl. Phys. **82**, 3028 (1997), and references therein.
[7] J. R. Clem and M. W. Coffey, Phys. Rev. B **46**, 14662 (1992).
[8] S. J. Turneaure, A. A. Pesetski, and T. R. Lemberger, J. Appl. Phys. **83**, 4334 (1998).
[9] To find the penetration depth in the limit $J\rightarrow 0$ it does not matter which coil is used as the drive.
[10] In principle the trapped vortices could increase the apparent penetration depth due to their reversible motion in pinning wells, cf. Ref. 7 above. In HTS films at the low fields involved here this effect is negligible.
[11] B. A Willemsen, K. E. Kihlstrom, T. Dahm, and D. J. Scalapino, IEEE Trans. Appl. Supercond. (to be published).
[12] This dependence is predicted for a *d*-wave superconductor at very low temperatures, cf. D. Xu, S. K. Yip, and J. A. Sauls, Phys. Rev. B **51**, 16233 (1995). At the temperature of the measurements reported here the intrinsic dependence of λ should be purely quadratic for either *s*- or *d*-wave superconductivity.